Course Curriculum

Doctoral Program
School of Mathematical Sciences

Applicable from the Academic Year 2018-19

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

BHUBANESWAR

SCHOOL OF MATHEMATICAL SCIENCES

COURSE STRUCTURE FOR DOCTORAL PROGRAM

After joining NISER, a Ph.D. student in the School of Mathematical Sciences has to undergo two semesters of course work in the first year. The course structure for these two semesters is described below:

Course No.	Credits	Course Name	Semester
MA601	8	Algebra-I	I
MA603	8	Analysis-I	I
MA605	8	Topology-I	I
MA696	–	Research Methodology	I
MA697	4	Seminar Course-I	I
MA607	8	Complex Analysis	II
MA***	8	Elective-I	II
MA***	8	Elective-II	II
MA698	4	Seminar Course-II	II
MA699	4	Seminar Course-III	II

Any two of the following three courses can be chosen by a student as Elective-I and Elective-II:

Course No.	Credits	Course Name	Semester
MA602	8	Algebra-II	II
MA604	8	Analysis-II	II
MA606	8	Topology-II	II

For the seminar course in a given semester, each student will choose a faculty from the school as his/her seminar course mentor. The topics of the seminar courses can be chosen from the list provided below or any other topic in advanced mathematics suggested by the mentor. Research methodology is a compulsory course with no credits. The total number credits required for the course work in the first year is $8 \times 6 + 4 \times 3 = 60$.

Having completed the first year course work successfully, a student will appear for Oral General Comprehensive Examination (OGCE) conducted by the monitoring committee. OGCE will be based on all the courses that a student has taken in the first year and will be conducted before the registration of third semester. The process of assigning Ph.D. thesis advisor/guide starts only after the successful completion of OGCE and it has to be done within a month from the date of completion of OGCE.

A doctoral committee for each student is then constituted by the Dean, Academic Affairs as per HBNI guidelines once the topic of research and thesis advisor has been identified. If the doctoral committee so desires, a student can take more courses related to his/her topic of research. The doctoral committee will meet at least once in every year to monitor the progress of a student. The student has to give one seminar every year before the doctoral committee. The thesis advisor, who is the convener of the doctoral committee, should submit the annual progress report of the student to the Dean, Academic Affairs every year before registration of odd/even semester depending on the joining date of the student and a copy of the report should be sent to the PGCS Convener for further reference.

LIST OF SEMINAR COURSES

Representations of Finite Groups	Differential Equations
Commutative Algebra	Partial Differential Equations
Advanced Linear Algebra	Advanced Partial Differential Equations
Algebraic Geometry	Ergodic Theory
Finite Fields	Probability Theory-I
Classical Groups	Probability Theory-II
Introduction to Number Theory	Advanced Probability
Advanced Number Theory	Mathematical Foundations for Finance
Algebraic Number Theory	Brownian Motion and Stochastic Calculus
Analytic Number Theory	Randomized Algorithms and Probabilistic Methods
Modular Forms of One Variable	Introduction to Stochastic Processes
Elliptic Curves	Statistical Inference I
Discrete Mathematics	Statistical Inference II
Graph Theory	Multivariate Statistical Analysis
Combinatorics and Graph Theory	Regression Analysis
Random graphs	Time Series Analysis
Algebraic Graph Theory	
Mathematical Logic	
Theory of Algorithms	
Cryptology	
Information and Coding Theory	
Theory of Computation	
Designs and Codes	
Algebraic Combinatorics	
Foundations of Cryptography	
Algebraic Computation	
Incidence Geometry	
Algebraic Topology	
Differential Geometry	
Differentiable Manifolds and Lie Groups	
Introduction to Manifolds	
Measure Theory	
Advanced Functional Analysis	
Nonlinear Analysis	
Optimization Theory	
Operator Algebras	
Operator Theory	
Harmonic Analysis	
Abstract Harmonic Analysis	
Lie Algebras	
Lie Groups and Lie Algebras-I	
Lie Groups and Lie Algebras-II	
Representations of Linear Lie Groups	
Harmonic Analysis on Compact Groups	

Program Outcome: PhD in Mathematics

Upon completing the PhD degree in the field of Mathematics, students have/capable of:

- A solid understanding of graduate level algebra, analysis and topology.
- Using their mathematical knowledge to tackle research problems.
- Identifying unsolved yet relevant problems in a specific field.
- Undertaken original research on a particular topic.
- Communicate mathematics accurately and effectively in both written and oral form.
- Conducting scholarly or professional activities in an ethical manner.

DETAILED SYLLABUS

Theory Courses

(MA601): (Algebra-I), Credits - 8
(3 Lectures + 1 Tutorial)
Outcome: Students will learn basic properties of groups, rings, and modules and will be able to use these algebraic structures to solve research problems.

Contents: Group Theory: Dihedral groups, Permutation groups, Group actions, Sylow's theorems, Simplicity of the alternating groups, Direct and semidirect products, Solvable groups, Nilpotent groups, Jordan Holder Theorem, free groups.
Ring Theory: Properties of Ideals, Chinese remainder theorem, Field of fractions, Euclidean domains, Principal ideal domains, Unique factorization domains, Polynomial Rings, Irreducibility criteria, Matrix rings.
Module Theory: Examples, quotient modules, isomorphism theorems, Generation of modules, free modules, tensor products of modules, Exact sequences - Projective, Injective and Flat modules.

Reference reading materials:

1. D. S. Dummit and R. M. Foote, Abstract Algebra. John Wiley & Sons, 2004.
2. T. W. Hungerford, Algebra, Graduate Texts in Mathematics, 73, Springer, 1980.
3. M. Artin, Algebra, Prentice Hall, 1991.
4. N. Bourbaki, Algebra, Springer, 1989.
5. C Musili, Introduction to Rings and Modules, Narosa Publishing House.
6. N. S. Gopalakrishnan, University Algebra, New Age International

(MA602): (Algebra-II), Credits - 8
(3 Lectures + 1 Tutorial)
Outcome: Students will learn basic properties of fields and Galois theory and will be able to use these results to solve other mathematical problems.

Contents: Linear Algebra: Matrix of a Linear transformation, dual vector spaces, determinants, Tensor algebras, Symmetric algebras, Exterior algebras,
Modules over PIDs: Basic theory, Structure theorem for finitely generated abelian groups, Rational and Jordan canonical forms.
Field Theory: Algebraic extensions, Splitting fields, Algebraic closures, Separable and Inseparable extensions, Cyclotomic polynomials and extensions, Galois extensions, Fundamental Theorem of Galois theory, Finite fields, Composite extensions, Simple extensions, Cyclotomic extensions and Abelian extensions over rational field, Galois groups of polynomials, Fundamental theorem of algebra, Solvable and Radical extensions, Computation of Galois groups over rational field.

Reference reading materials:

1. D. S. Dummit and R. M. Foote, Abstract Algebra. John Wiley & Sons, 2004.
2. T. W. Hungerford, Algebra, Graduate Texts in Mathematics, 73, Springer, 1980.
3. M. Artin, Algebra, Prentice Hall, 1991.
4. T. T. Moh: Algebra, World Scientific, 1992
5. N. Bourbaki, Algebra, Springer, 1989.

(MA603): (Analysis-I), Credits - 8
(3 Lectures + 1 Tutorial)

Outcome: Upon successful completion of the course, students will be familiar with various advanced concepts and techniques from functional analysis, measure theory and harmonic analysis (on the real line).

Contents: Spaces of functions: Continuous functions on locally compact spaces, Stone-Weierstrass theorems, Ascoli-Arzela Theorem. Review of Measure theory: Sigma-algebras, measures, construction and properties of the Lebesgue measure, non-measurable sets, measurable functions and their properties. Integration: Lebesgue Integration, various limit theorems, comparison with the Riemann Integral, Functions of bounded variation and absolute continuity. Measure spaces: Signed-measures, Radon-Nikodym theorem, Product spaces, Fubini's thoerem (without proof) and its applications. Lp-spaces: Holder and Minkowski inequalities, completeness, Convolutions, Approximation by smooth functions. Fourier analysis: Fourier Transform, Inverse Fourier transform, Plancherel Theorem for Real numbers.

Reference reading materials:

1. D. S. Bridges, Foundations of Real and Abstract Analysis, GTM series, Springer Verlag 1997.
2. G. B. Foland, Real Analysis: Modern Techniques and Their Applications (2nd ed.), Wiley-Interscience/John Wiley Sons, Inc., 1999.
3. P. R. Halmos, Measure Theory, Springer-Verlag, 1974.
4. H. L. Royden, Real Analysis, Macmillan 1988.
5. W. Rudin, Real and Complex Analysis, TMH Edition, Second Edition, New-York, 1962.
6. Elliott H. Lieb and Michael Loss , Analysis, American Mathematical Society, 2001

(MA604): (Analysis-II), Credits - 8

(3 Lectures + 1 Tutorial)

Outcome: Upon successful completion of the course, students will aware of various properties of norm linear vector spaces and topological vector spaces. They will also learn various properties of linear transformations defined on these norm liner spaces and topological vector spaces.

Contents: Banach spaces: Review of Banach spaces, Hahn-Banach Theorem and its applications, Baire Category theorem and its applications like Closed graph theorem, Open mapping theorem. Topological Vector spaces: Weak and weak* topologies, locally convex topological vector spaces. Hilbert spaces: Review of Hilbert spaces and operator Theory, Compact operators, Schauder's theorem on the spectral theory of compact operators. Banach algebras: Elementary properties, Resolvent and spectrum, Spectral radius formula, Ideals and homomorphisms, Gelfand transforms, Gelfand theorem for commutative Banach algebras.

Reference reading materials:

1. D. S. Bridges, Foundations of Real and Abstract Analysis, GTM series, Springer Verlag 1997.
2. G. B. Foland, Real Analysis: Modern Techniques and Their Applications (2nd ed.), Wiley-Interscience/John Wiley Sons, Inc., 1999.
3. G. K. Pederson, Analysis NOW, GTM series, Springer-Verlag, 1991.
4. W. Rudin, Real and Complex Analysis, TMH Edition, Second Edition, New-York, 1962.
5. W. Rudin, Functional Analysis, TMH Edition, 1974.
6. K. Yosida, Functional Analysis, Springer-Verlag 1968.

(MA605): (Topology-I), Credits - 8
(3 Lectures + 1 Tutorial)
Outcome: Upon successful completion of the course, students will aware of various properties of topological space and various properties of functions on topological spaces. The students also learn continuous maps between topological spaces, product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, separation axioms, Tychonoff spaces, Urysohns lemma and metrization theorem. Furthermore, the student will learn various properties of functions for several variable.

Contents: Topological spaces, Continuous maps between topological spaces, product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, separation axioms, Tychonoff spaces, Urysohn's lemma and metrization theorem
Differentiable functions on $\mathbb{R}^n$, Jacobian criteria, Taylor's theorem, Inverse function theorem, Implicit function theorem, Maxima-minima, Lagrange multiplier.

Reference reading materials:

1. Armstrong, *Basic Topology,* Springer, 1983
2. Munkres, *Topology,* Pearson Education, 2005.
3. J. Dugundji, Topology
4. J. J. Duistermaat, J. A. C. Kolk: Multidimensional Real Analysis I: Differentiation
5. K. Janich, Topology, Springer
6. John L Kelley: General Topology (free download: https://archiveorg/details/GeneralTopology)
7. F. Simmons: Introduction to Topology and Modern Analysis
8. S.Kumaresan: A Course in Differential Geometry and Lie Groups,TRIM series
9. T. M. Apostol: Calculus: Multi-Variable Calculus and Linear Algebra With Applications to Differential Equations And Probability- Vol 2.

(MA606): (Topology-II), Credits - 8
(3 Lectures + 1 Tutorial)
Outcome: Upon successful completion of the course, students will learn covering spaces, homotopy theory and several concepts of topological invariants, namely, fundamental groups, homology groups.

Contents: Homotopy Theory: Fundamental groups and its functorial properties, examples, Van- Kampen Theorem, Computation of fundamental group of S^1.

Covering spaces: Covering spaces, Computation of fundamental groups using cover- ings. The classification of covering spaces. Deck transformations.

Simply connected spaces: Simply connected spaces-Universal covering spaces of locally simply connected and pathwise connected spaces. - Universal covering group of connected subgroups of General Linear groups.

Homology groups: Affine spaces, simplexes and chains - Homology groups - Properties of Homology groups. - Chain Complexes, Relation Between one dimensional Homotopy and Homology groups. Computation of Homology groups S^n, Brouwer's fixed point theorem.

Reference reading materials:

1. Armstrong, Basic Topology, Springer, 1983
2. Greenberg & Harper, Algebraic Topology: A First Course, Addition Wesley, 1984.
3. Munkres, Topology, Pearson Education, 2005. 1974

(MA607): (Complex Analysis), Credits - 8
(3 Lectures + 1 Tutorial)
Outcome: Students will learn some important theorems in complex anal- ysis such as Riemann

mapping theorem, Weirstrass factorization theorem, Runges theorem, Hardamard factorization theorem, Little Picards theorem and Great Picards theorem. They will also learn some basic techniques of harmonic functions and characterization of Dirichlet Region. These results are very useful in many branches of mathematics such as Number Thoery, Differential Geometry, Operator theory, Partal Differential Equations etc.

Contents: Review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues; Normal families, Arzela-Ascoli theorem, Riemann mapping theorem; Weierstrass factorization theorem, Runges theorem, Mittag-Lefflers theorem; Hadamard factorization theorem, Analytic Continuation, Gamma and Zeta functions.

Reference reading materials:

1. L. V. Ahlfors, Complex Analysis, Tata McGraw-Hill, 2013.
2. J. B. Conway, "Functions of one complex variable", Second edition. Graduate Texts in Mathematics, 11. Springer-Verlag, New York-Berlin, 1978.
3. R. Narasimhan and Y. Nievergelt, "Complex analysis in one variable", Second edition. Birkhuser Boston, Inc., Boston, MA, 2001.
4. W. Rudin, Real and Complex Analysis, Tata McGraw-Hill, 2013.
5. Wolfgang Fischer, Ingo Lieb, A Course in Complex Analysis: From Basic Results to Advanced Topics, Springer, 2012
6. Eberhard Freitag, Rolf Busam, Complex Analysis, Springer, 2005
7. Stein and Shakarchi, Complex Analysis, Princeton University Press, 2003
8. Gamelin, Complex Analysis, Springer, 2000

(M699): (Research Methodology), Credits - 0

The actual contents of the course are left to the instructor, but it should include topics on ethics in publishing, plagiarism, how to write research articles etc. This course has only pass or fail option.

Seminar Courses

(Representations of Finite Groups)

Group representations, Maschke's theorem and completely reducibility, Characters, Inner product of Characters, Orthogonality relations, Burnside's theorem, induced characters, Frobenius reciprocity, induced representations, Mackey's Irreducibility Criterion, Character table of some well-known groups, Representation theory of the symmetric group: partitions and tableaux, constructing the irreducible representations.

Reference reading materials:

1. G. James, M. Liebeck, "Representations and Characters of Groups", Cambridge University Press, 2010.
2. J. L. Alperin, R. B. Bell, "Groups and Representations", Graduate Texts in Mathematics 162, Springer, 1995.
3. B. Steinberg, "Representation Theory of Finite Groups", Universitext, Springer, 2012.
4. J-P. Serre, "Linear Representations of Finite Groups", Graduate Texts in Mathematics 42, Springer-Verlag, 1977.
5. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.
6. Martin Isaacs, Character theory of finite groups, Dover, 1976

(Commutative Algebra)

Commutative rings, ideals, operations on ideals, prime and maximal ideals, nilradicals, Jacobson radicals, extension and contraction of ideals, Modules, free modules, projective modules, exact sequences, tensor product of modules, Restriction and extension of scalars, localization and local rings, extended and contracted ideals in rings of fractions, Noetherian modules, Artinian modules, Primary decompositions and associate primes, Integral extensions, Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals, Completion, Dimension theory.

Reference reading materials:

1. M. F. Atiyah, I. G. Macdonald, "Introduction to Commutative Algebra", Addison-Wesley Publishing Co., 1969.
2. R. Y. Sharp, "Steps in Commutative Algebra", London Mathematical Society Student Texts, 51. Cambridge University Press, 2000.
3. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.
4. N. S. Gopalakrishnan, University Algebra, New Age International

(Advanced Linear Algebra)

Rational and Jordan canonical forms, Inner product spaces, Unitary and Normal operators, Forms on inner product spaces, Spectral theorems, Bilinear forms, Matrix decomposition theorems, Courant- Fischer minimax and related theorems, Nonnegative matrices, Perron-Frobenius theory, Generalized inverse, Matrix Norm, Perturbation of eigenvalues.

Reference reading materials:

1. R. A. Horn, C. R. Johnson, "Matrix Analysis", Cambridge University Press, 2010.
2. K. Hoffman, R. Kunze, "Linear Algebra", Prentice-Hall of India, 2012.
3. S. Roman, "Advanced Linear Algebra", Graduate Texts in Mathematics 135, Springer, 2008.

(Algebraic Geometry)

Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert's Nullstellensatz, Affine and Projective varieties, Zariski Topology, Rational functions and morphisms, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Riemann-Roch for curves, Line bundles on Projective spaces.

Reference reading materials:

1. K. Hulek, “Elementary Algebraic Geometry”, Student Mathematical Library 20, American Mathematical Society, 2003.
2. I. R. Shafarevich, “Basic Algebraic Geometry 1: Varieties in Projective Space”, Springer, 2013.
3. J. Harris, “Algebraic geometry”, Graduate Texts in Mathematics 133, Springer-Verlag, 1995.
4. M. Reid, “Undergraduate Algebraic Geometry”, London Mathematical Society Student Texts 12, Cambridge University Press, 1988.
5. K. E. Smith et. al., “An Invitation to Algebraic Geometry”, Universitext, Springer-Verlag, 2000.
6. R. Hartshorne, “Algebraic Geometry”, Graduate Texts in Mathematics 52, Springer-Verlag, 1977.

(Finite Fields)

Structure of finite fields: characterization, roots of irreducible polynomials, traces, norms and bases, roots of unity, cyclotomic polynomial, representation of elements of finite fields, Wedderburn’s theorem; Polynomials over finite field: order of polynomials, primitive polynomials, construction of irreducible polynomials, binomials and trinomials, factorization of polynomials over small and large finite fields, calculation of roots of polynomials; Linear recurring sequences: LFSR, characteristic polynomial, minimal polynomial, characterization of linear recurring sequences, Berlekamp-Massey algorithm; Applications of finite fields: Applications in cryptography, coding theory, finite geometry, combinatorics.

Reference reading materials:

1. R. Lidl, H. Neiderreiter, “Finite Fields”, Cambridge university press, 2000.
2. G. L. Mullen, C. Mummert, “Finite Fields and Applications”, American Mathematical Society, 2007.
3. A. J. Menezes et. al., “Applications of Finite Fields”, Kluwer Academic Publishers, 1993.
4. Z-X. Wan, “Finite Fields and Galois Rings”, World Scientific Publishing Co., 2012.

(Classical Groups)

General and special linear groups, bilinear forms, Symplectic groups, symmetric forms, quadratic forms, Orthogonal geometry, orthogonal groups, Clifford algebras, Hermitian forms, Unitary spaces, Unitary groups.

Reference reading materials:

1. L. C. Grove, “Classical Groups and Geometric Algebra”, Graduate Studies in Mathematics 39, American Mathematical Society, 2002.
2. E. Artin, “Geometric Algebra”, John Wiley & sons, 1988.
3. Curtis, 'Matrix groups', Springer, 1979

(Introduction to Number Theory)

The Fundamental Theorem of Arithmetic, Distribution of prime numbers, Congruences, Chinese remainder theorem, Congruences with prime-power modulus, Fermat’s little theorem, Wilson’s theorem, Euler function and its applications, Group of units, Primitive roots, Quadratic residues and Quadratic reciprocity law, Arithmetic functions, Mobius Inversion formula, Dirichlet product, Sum of squares, Introduction to Zeta function and Dirichlet Series.

Reference reading materials:

1. G. H. Hardy & E. M. Wright: An Introduction to the Theory of Numbers (Oxford).
2. J. A. Jones and J. M. Jones: Elementary Number Theory (Springer).
3. I. Niven, H. S. Zuckerman & H. L. Montgomery: The Theory of Numbers (Wiley)
4. T. M. Apostol: Introduction to Analytic Number Theory (Springer)

(Advanced Number Theory)

Review of Finite fields, Gauss Sums and Jacobi Sums, Cubic and biquadratic reciprocity, Polynomial equations over finite fields, Theorems of Chevally and Warning, Quadratic forms over prime fields. Ring of p-adic integers, Field of p-adic numbers, completion, p-adic equations, Hensel's lemma, Hilbert symbol, Quadratic forms with p-adic coefficients. Dirichlet series: Abscissa of convergence and absolute convergence, Riemann Zeta function and Dirichlet L-functions. Dirichlet's theorem on primes in arithmetic progression. Functional equation and Euler product for L-functions. Modular Forms and the Modular Group, Eisenstein series, Zeros and poles of modular functions, Dimensions of the spaces of modular forms, The j-invariant L-function associated to modular forms, Ramanujan τ function.

Reference reading materials:

1. J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
2. K. Ireland, M. Rosen, "A Classical Introduction to Modern Number Theory", Graduate Texts in Mathematics 84, Springer-Verlag, 1990.
3. H. Hasse, "Number Theory", Classics in Mathematics, Springer-Verlag, 2002.
4. W. Narkiewicz, "Elementary and Analytic Theory of Algebraic Numbers", Springer Monographs in Mathematics, Springer-Verlag, 2004.
5. F. Q. Gouvêa, "p-adic Numbers", Universitext, Springer-Verlag, 1997.
6. Borevich and Shafarevich, Number Theory, Academic press, 1986.

(Algebraic Number Theory)

Number Fields and Number rings, prime decomposition in number rings, Dedekind domains, Ideal class group, Galois theory applied to prime decomposition, Gauss reciprocity law, Cyclotomic fields and their ring of integers, finiteness of ideal class group, Dirichlet unit theorem, valuations and completions of number fields, Dedekind zeta function and distribution of ideal in a number ring.

Reference reading materials:

1. D. A. Marcus, "Number Fields", Universitext, Springer-Verlag, 1977.
2. G. J. Janusz, "Algebraic Number Fields", Graduate Studies in Mathematics 7, American Mathematical Society, 1996.
3. S. Alaca, K. S. Williams, "Introductory Algebraic Number Theory", Cambridge University Press, 2004.
4. S. Lang, "Algebraic Number Theory", Graduate Texts in Mathematics 110, Springer-Verlag, 1994.
5. A. Frohlich, M. J. Taylor, "Algebraic Number Theory", Cambridge Studies in Advanced Mathematics 27, Cambridge University Press, 1993.
6. J. Neukirch, "Algebraic Number Theory", Springer-Verlag, 1999.
7. Ram Murty and Esmonde, 'Problems in algebraic number theory', Springer, 1999.

(Analytic Number Theory)

Arithmetic functions, Averages of arithmetical functions, Distribution of primes, finite abelian groups and characters, Gauss sums, Dirichlet series and Euler products, Reimann Zeta function, Dirichlet L-functions, Analytic proof of the prime number theorem, Dirichlet Theorem on primes in arithmetic progression.

Reference reading materials:

1. T. M. Apostol, "Introduction to Analytic Number Theory", Springer International Student Edition, 2000.
2. K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, 1968.
3. H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, 2004.
4. Ram Murty, Problem book in analytic number theory, Springer, 2008.

(Modular Forms of One Variable)

$SL_2(\mathbb{Z})$ and its congruence subgroups, Modular forms for $SL_2(\mathbb{Z})$, Modular forms for congruence subgroups, Modular forms and differential operators, Hecke theory, L-series, Theta functions and transformation formula.

Reference reading materials:

1. J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.

2. N. Koblitz, “Introduction to Elliptic Curves and Modular Forms”, Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Bruinier, G. van der Geer, G. Harder, D. Zagier, “The 1-2-3 of Modular Forms”, Universitext, Springer-Verlag, 2008.
4. F. Diamond, J. Shurman, “A First Course in Modular Forms”, Graduate Texts in Mathematics 228, Springer-Verlag, 2005.
5. S. Lang, “Introduction to Modular Forms”, Springer-Verlag, 1995.
6. G. Shimura, “Introduction to the Arithmetic Theory of Automorphic Forms”, Princeton University Press, 1994.
7. Ram Murty, Problem in the theory of modular forms, Hindustan book agency, 2015

(Elliptic Curves)

Congruent numbers, Elliptic curves, Elliptic curves in Weierstrass form, Addition law, Mordell–Weil Theorem, Points of finite order, Points over finite fields, Hasse-Weil L-function and its functional equation, Complex multiplication.

Reference reading materials:

1. J. H. Silverman, J. Tate, “Rational Points on Elliptic Curves”, Undergraduate Texts in Mathematics, Springer-Verlag, 1992.
2. N. Koblitz, “Introduction to Elliptic Curves and Modular Forms”, Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Silverman, “The Arithmetic of Elliptic Curves”, Graduate Texts in Mathematics 106, Springer, 2009.
4. A. W. Knapp, “Elliptic Curves”, Mathematical Notes 40, Princeton University Press, 1992.
5. J. H. Silverman, “Advanced Topics in the Arithmetic of Elliptic Curves”, Graduate Texts in Mathematics 151, Springer-Verlag, 1994.
6. Cassels, Lectures on elliptic curves, London Mathematical Society, 1991.

(Discrete Mathematics)

Combinatorics:Counting principles, Generating functions, Recurrence relation, Polyas enumeration theory, partially ordered sets.
Graph Theory:Graphs, Trees, Blocks, Connectivity, Eulerian and Hamiltonian graphs, Planer graphs, Graph colouring.
Design Theory: Block Designs, Balanced incomplete block design, Difference sets and Automorphism, Latin squares, Hadamard matrices, Projective planes, Generalized quadrangles.
Algorithm:Algorithm, Asymptotic analysis, Complexity hierarchy, NP-complete problems.

Reference reading materials:

1. F. Roberts and B. Tesman: Applied Combinatorics. Pearson Education, 2005.
2. M. Aigner, A course in Enumeration, Springer.
3. R. P. Stanley, Enumerative Combinatorics, Cambridge University Press.
4. F. Harary, Graph Theory, Narosa Publishing House.
5. G.A. Bondy and U.S.R. Murty: Graph Theory. Springer, 2008.
6. W. D. Wallis, Introduction to Combinatorial Designs, Chapman & Hall/CRC
7. D. R. Stinson and D. Stinson, Combinatorial Designs: Construction and Analysis, Springer.
8. Thomas Cormen, Charles Leiserson, Ronald Rivest: Introduction to Algorithms. PHI, 1998.
9. K. H. Rosen, Discrete Mathematics and its Applications, McGraw Hill Education; 4th Revised edition edition (1 January 1999)

(Graph Theory)

Basic definitions, Eulerian and Hamiltonian graphs, Planarity, Colourability, Four colour problem, Matching and Halls marriage theorem, Max-flow Min-cut theorem, Ramsey theory, Line graphs, Enumeration, Digraphs. Matroids, Groups and Graphs, Matrices and graphs, Eigenvalues of graphs, The Laplacian of a graph, Strongly regular graphs.

Reference reading materials:

1. D. B. West, Introduction to Graph Theory, Prentice Hall of India.
2. F. Harary, Graph Theory, Narosa Publishing House.
3. B. Bollobas, Extremal Graph Theory , Dover Publications.
4. R. Diestel, Graph Theory, Springer International Edition.
5. G. A. Bondy and U. S. R. Murty, Graph Theory, Springer
6. C. Godsil and G. Royle, Algebraic Graph Theory, Springer International Edition.

(Combinatorics and Graph Theory)

Pigeonhole principle, Counting principles, Binomial coefficients, Principles of inclusion and exclusion, recurrence relations, generating functions, Catalan numbers, Stirling numbers, Partition numbers, Schroder numbers.[25 lectures]
Graphs, subgraphs, graph isomorphisms, Hamilton cycles, Euler tours, directed graphs, matching, Tutte's theorem, Menger's theorem, planar graphs, Kuratowski's theorem, graph colourings, network flows, max-flow min-cut theorem, Ramsey theory for graphs, Matrices associated with graphs: Incidence matrix, Adjacency matrix, Laplacian matrix.[25 lectures]

Reference reading materials:

1. R. A. Brualdi, Introductory Combinatorics, Pearson Prentice Hall, 2010.
2. J. H. van Lint, R. M. Wilson, A Course in Combinatorics, Cambridge University Press, 2001.
3. R. P. Stanley, Enumerative Combinatorics Vol. 1, Cambridge Studies in Advanced Mathematics, 49, Cambridge University Press, 2012.
4. R. Diestel, Graph Theory, Graduate Texts in Mathematics, 173, Springer, 2010.
5. B. Bollobas, Modern Graph Theory, Graduate Texts in Mathematics, 184, Springer-Verlag, 1998.
6. J. A. Bondy, U. S. R. Murty, Graph Theory, Graduate Texts in Mathematics, 244, Springer, 2008.

(Random Graphs)

Models of random graphs and of random graph processes; illustrative examples; random regular graphs, configuration model; appearance of the giant component small subgraphs; long paths and Hamiltonicity; coloring problems; eigenvalues of random graphs and their algorithmic applications; pseudo-random graphs.

Reference reading materials:

1. N. Alon, J. H. Spencer, "The Probabilistic Method", John Wiley & Sons, 2008
2. B. Bollobás, "Random Graphs", Cambridge Studies in Advanced Mathematics 73, Cambridge University Press, 2001.
3. S. Janson, T. Luczak, A. Rucinski, "Random Graphs", Wiley-Interscience, 2000.
4. R. Durrett, "Random Graph Dynamics", Cambridge University Press, 2010.
5. J. H. Spencer, "The Strange Logic of Random Graphs", Springer-Verlag, 2001.

(Algebraic Graph Theory)

Adjacency matrix of a graph and its eigenvalues, Spectral radius of graphs, Regular graphs and Line graphs, Strongly regular graphs, Cycles and Cuts, Laplacian matrix of a graph, Algebraic connectivity, Laplacian spectral radius of graphs, Distance matrix of a graph, General properties of graph automorphisms, Transitive and Arc-tranisitve graphs, Symmetric graphs.

Reference reading materials:

1. N. Biggs, "Algebraic Graph Theory", Cambridge University Press, 1993.
2. C. Godsil, G. Royle, "Algebraic Graph Theory", Graduate Texts in Mathematics 207, Springer-Verlag, 2001.
3. R. B. Bapat, "Graphs and Matrices", Universitext, Springer, Hindustan Book Agency, New Delhi, 2010.

(Mathematical Logic)

Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem. First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g., theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godels first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

Reference reading materials:

1. J. R. Shoenfield, "Mathematical logic", Addison-Wesley Publishing Co., 1967.
2. E. Mendelson, "Introduction to Mathematical Logic", Chapman & Hall, 1997.
3. Barnes and Mack, An algebraic introduction to mathematical logic, Springer.

(Theory of Algorithms)

Algorithm analysis: asymptotic notation, probabilistic analysis; Data Structure: stack, queues, linked list, hash table, binary search tree, red-black tree; Sorting: heap sort, quick sort, sorting in linear time; Algorithm design: divide and conquer, greedy algorithms, dynamic programming; Algebraic algorithms: Winograd's and Strassen's matrix multiplication algorithm, evaluation of polynomials, DFT, FFT, efficient FFT implementation; Graph algorithms: breadth-first and depth-first search, minimum spanning trees, single-source shortest paths, all-pair shortest paths, maximum flow; NP-completeness and approximation algorithms.

Reference reading materials:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, "The Design and Analysis of Computer Algorithms", Addison-Wesley Publishing Co., 1975.
2. T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, "Introduction to Algorithms", MIT Press, Cambridge, 2009.
3. E. Horowitz, S. Sahni, "Fundamental of Computer Algorithms", Galgotia Publication, 1987.
4. D. E. Knuth, "The Art of Computer Programming Vol. 1, Vol. 2, Vol 3", Addison-Wesley Publishing Co., 1997, 1998, 1998.

(Cryptology)

Overview: Cryptography and cryptanalysis, some simple cryptosystems (e.g., shift, substitution, affine, knapsack) and their cryptanalysis, classification of cryptosystems, classification of attacks; Information Theoretic Ideas: Perfect secrecy, entropy; Secret key cryptosystem: stream cipher, LFSR based stream ciphers, cryptanalysis of stream cipher (e.g., correlation attack, algebraic attacks), block cipher, DES, linear and differential cryptanalysis, AES; Public-key cryptosystem: Implementation and cryptanalysis of RSA, ElGamal public-key cryptosystem, Discrete logarithm problem, elliptic curve cryptography; Data integrity and authentication: Hash functions, message authentication code, digital signature scheme, ElGamal signature scheme; Secret sharing: Shamir's threshold scheme, general access structure and secret sharing.

Reference reading materials:

1. D. R. Stinson, "Cryptography: Theory And Practice", Chapman & Hall/CRC, 2006.
2. A. J. Menezes, P. C. van Oorschot, S. A. Vanstone, "Handbook of Applied Cryptography", CRC Press, 1997.
3. Koblitz, Number theory and cryptography, Springer, 1987.

(Information and Coding Theory)

Information Theory: Entropy, Huffman coding, Shannon-Fano coding, entropy of Markov process, channel and mutual information, channel capacity;
Error correcting codes: Maximum likelihood decoding, nearest neighbour decoding, linear codes, generator matrix and parity-check matrix, Hamming bound, Gilbert-Varshamov bound, binary Hamming codes, Plotkin bound, nonlinear codes, Reed-Muller codes, Cyclic codes, BCH codes, Reed-Solomon codes, Algebraic codes.

Reference reading materials:

1. R. W. Hamming, "Coding and Information Theory", Prentice-Hall, 1986.
2. N. J. A. Sloane, F. J. MacWilliams, "Theory of Error Correcting Codes", North-Holland Mathematical Library 16, North-Holland, 2007.
3. S. Ling, C. Xing, "Coding Theory: A First Course", Cambridge University Press, 2004.
4. V. Pless, "Introduction to the Theory of Error-Correcting Codes", Wiley-Interscience Publication, John Wiley & Sons, 1998.
5. S. Lin, "An Introduction to Error-Correcting Codes", Prentice-Hall, 1970.

(Theory of Computation)

Automata and Language Theory: Finite automata, regular expression, pumping lemma, context free grammar, context free languages, Chomsky normal form, push down automata, pumping lemma for CFL; Computability: Turing machines, Churh-Turing thesis, decidability, halting problem, reducibility, recursion theorem; Complexity: Time complexity of Turing machines, Classes P and NP, NP completeness, other time classes, the time hierarchy.

Reference reading materials:

1. J. E. Hopcroft, R. Motwani, J. D. Ullman, "Introduction to Automata Theory, Languages, and Computation", Addison-Wesley, 2006.
2. H. Lewis, C. H. Papadimitriou, "Elements of the Theory of Computation", Prentice-Hall, 1997.
3. M. Sipser, "Introduction to the Theory of Computation", PWS Publishing, 1997.

(Designs and Codes)

Incidence structures, affine planes, translation plane, projective planes, conics and ovals, blocking sets. Introduction to Balanced Incomplete Block Designs (BIBD), Symmetric BIBDs, Difference sets, Hadamard matrices and designs, Resolvable BIBDs, Latin squares. Basic concepts of Linear Codes, Hamming codes, Golay codes, Reed-Muller codes, Bounds on the size of codes, Cyclic codes, BCH codes, Reed-Solomon codes.

Reference reading materials:

1. G. Eric Moorhouse, "Incidence Geometry", 2007 (available online).
2. Douglas R. Stinson, "Combinatorial Designs", Springer-Verlag, New York, 2004.
3. W. Cary Huffman, V. Pless, "Fundamentals of Error-correcting Codes", Cambridge Uinversity Press, Cambridge, 2003.

(Algebraic Combinatorics)

Catalan Matrices and Orthogonal Polynomials, Catalan Numbers and Lattice Paths, Combinatorial Interpretation of Catalan Numbers, Symmetric Polynomials and Functions, Schur Functions, Jacobi-Trudi identity, RSK Algorithm, Standard Tableaux, Young diagrams and q-binomial coefficients, Plane Partitions, Group actions on boolean algebras, Enumeration under group action, Walks in graphs, Cubes and the Radon transform, Sperner property, Matrix-Tree Theorem.

Reference reading materials:

1. R. P. Stanley, "Algebraic Combinatorics", Undergraduate Texts in Mathematics, Springer, 2013.

2. M. Aigner, “A Course in Enumeration”, Graduate Texts in Mathematics 238, Springer, 2007.
3. R. P. Stanley, “Enumerative Combinatorics Vol. 2”, Cambridge Studies in Advanced Mathematics 62, Cambridge University Press, 1999.

(Foundations of Cryptography)

Introduction to cryptography and computational model, computational difficulty, pseudorandom generators, zero-knowledge proofs, encryption schemes, digital signature and message authentication schemes, cryptographic protocol.

Reference reading materials:

1. O. Goldreich, “Foundations of Cryptography - Vol. I and Vol. II”, Cambridge University Press, 2001, 2004.
2. S. Goldwasser, Mihir Bellare, “Lecture Notes on Cryptography”, 2008, available online from http://cseweb.ucsd.edu/ mihir/papers/gb.html

(Algebraic Computation)

Linear algebra and lattices: Asymptotically fast matrix multiplication algorithms, linear algebra algorithms, normal forms over fields, Lattice reduction; Solving system of non-linear equations: Gröbner basis, Buchberger's algorithms, Complexity of Gröbner basis computation; Algorithms on polynomials: GCD, Barlekamp-Massey algorithm, factorization of polynomials over finite field, factorization of polynomials over $\mathbb{Z}$ and $\mathbb{Q}$; Algorithms for algebraic number theory: Representation and operations on algebraic numbers, trace, norm, characteristic polynomial, discriminant, integral bases, polynomial reduction, computing maximal order, algorithms for quadratic fields; Elliptic curves: Implementation of elliptic curve, algorithms for elliptic curves.

Reference reading materials:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, “The Design and Analysis of Computer Algorithms”, Addison-Wesley Publishing Co., 1975.
2. H. Cohen, “A Course in Computational Algebraic Number Theory”, Graduate Texts in Mathematics 138, Springer-Verlag, 1993.
3. D. Cox, J. Little, D. O'shea, “Ideals, Varieties and Algorithms: An introduction to computational algebraic geometry and commutative algebra”, Undergraduate Texts in Mathematics, Springer-verlag, 2007.

(Incidence Geometry)

Definitions and Exampleas, projective planes, affine planes, projective spaces, affine spaces, collineations of projective and affine spaces, fundamental theorem of projective and affine spaces, polar spaces, generalized quadrangles, quadrics and quadratic sets.

Reference reading materials:

1. J. Ueberberg, “Foundations of Incidence Geometry”, Springer Monographs in Mathematics, Springer, 2011.
2. L. M. Batten, “Combinatorics of Finite Geometries”, Cambridge University Press, 1997.
3. E. E. Shult, “Points and Lines”, Universitext, Springer, 2011.
4. L. M. Batten, A. Beutelspacher, “The Theory of Finite Linear Spaces: Combinatorics of points and lines”, Cambridge University Press, 1993.
5. G. E. Moorhouse, “Incidence Geometry”, 2007, available online from http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf

(Algebraic Topology)

Homotopy Theory: Simply Connected Spaces, Covering Spaces, Universal Covering Spaces, Deck Transformations, Path lifting lemma, Homotopy lifting lemma, Group Actions, Properly discontinuous action, free groups, free product with amalgamation, Seifert-Van Kampen

Theorem, Borsuk-Ulam Theorem for sphere, Jordan Separation Theorem. Homology Theory: Simplexes, Simplicial Complexes, Triangulation of spaces, Simplicial Chain Complexes, Simplicial Homology, Singular Chain Complexes, Cycles and Boundary, Singular Homology, Relative Homology, Short Exact Sequences, Long Exact Sequences, Mayer-Vietoris sequence, Excision Theorem, Invariance of Domain.

Reference reading materials:

1. J. R. Munkres, "Topology", Prentice-Hall of India, 2013.
2. A. Hatcher, "Algebraic Topology", Cambridge University Press, 2009.
3. Fulton, Algebraic topology (Springer-Verlag GTM), 1995.
4. G. E. Bredon, "Topology and Geometry", Graduates Texts in Mathematics 139, Springer, 2009.

(Differential Geometry)

Parametrized curves in $\mathbb{R}^3$, length of curves, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frnet formula, curvatures, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, Umlaufsatz. Smooth vector fields on an open subset of $\mathbb{R}^3$, gradient vector field of a smooth function, vector field along a smooth curve, integral curve of a vectorfield. Existence theorem of an integral curve of a vector field through a point, maximal integral curve through a point.
Level sets, examples of surfaces in $\mathbb{R}^3$. Tangent spaces at a point. Vector fields on surfaces. Existence theorem of integral curve of a smooth vector field on a surface through a point. Existence of a normal vector of a connected surface. Orientation, Gauss map. The notion of geodesic on a surface. The existence and uniqueness of geodesic on a surface through a given point and with a given velocity vector thereof. Covariant derivative of a smooth vector field. Parallel vector field along a curve. Existence and uniqueness theorem of a parallel vector field along a curve with a given initial vector. The Weingarten map of a surface at a point, its self-adjointness property. Normal curvature of a surafce at a point in a given direction. Principal curvatures, first and second fundamental forms, Gauss curvature and mean curvature. Surface area and volume. Surfaces with boundary, local and global stokes theorem. Gauss-Bonnet theorem.

Reference reading materials:

1. A. Pressley, Elementary Differential Geometry, Springer (Indian reprint 2004)
2. J.A. Thorpe, Elementary topics in Differential Geometry, Springer (Indian reprint 2004).
3. B. ONeill, Elementary Differential Geometry, Academic Press (1997).

(Differentiable Manifolds and Lie Groups)

Review of Several variable calculus: Directional derivatives, Inverse Function Theorem, Implicit function Theorem, Level sets in $\mathbb{R}^n$, Taylor's theorem, Smooth function with compact support. Manifolds: Differentiable manifold, Partition of Unity, Tangent vectors, Derivative, Lie groups, Immersions and submersions, Submanifolds. Vector Fields: Left invariant vector fields of Lie groups, Lie algebra of a Lie group, Computing the Lie algebra of various classical Lie groups. Flows: Flows of a vector field, Taylor's formula, Complete vector fields. Exponential Map: Exponential map of a Lie group, One parameter subgroups, Frobenius theorem (without proof). Lie Groups and Lie Algebras: Properties of Exponential function, product formula, Cartan's Theorem, Adjoint representation, Uniqueness of differential structure on Lie groups. Homogeneous Spaces: Various examples and Properties. Coverings: Covering spaces, Simply connected Lie groups, Universal covering group of a connected Lie group. Finite dimensional representations of Lie groups and Lie algebras.

Reference reading materials:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.

2. S. Helgason, "Differential Geometry, Lie Groups and Symmetric Spaces", Graduate Studies in Mathematics 34, American Mathematical Society, 2001.
3. S. Kumaresan, "A Course in Differential Geometry and Lie Groups", Texts and Readings in Mathematics 22, Hindustan Book agency, 2002.
4. F. W. Warner, "Foundations of Differentiable Manifolds and Lie Groups", Graduate Texts in Mathematics 94, Springer-Verlag, 1983.

(Introduction to Manifolds)

Differentiable manifolds and maps: Definition and examples, Inverse and implicit function theorem, Submanifolds, immersions and submersions. The tangent and cotangent bundle: Vector bundles, (co)tangent bundle as a vector bundle, Vector fields, flows, Lie derivative. Differential forms and Integration: Exterior differential, closed and exact forms, Poincaré lemma, Integration on manifolds, Stokes theorem, De Rham cohomology.

Reference reading materials:

1. Michael Spivak, "A comprehensive introduction to differential geometry", Vol. 1, 3rd edition, 1999.
2. Frank Warner, "Foundations of differentiable manifolds and Lie groups", Springer-Verlag, 2nd edition, 1983.
3. John Lee, "Introduction to smooth manifolds", Springer Verlag, 2nd edition, 2013.
4. Louis Auslander and Robert E. MacKenzie, "Introduction to differentiable manifolds", Dover, 2nd edition, 2009.

(Measure Theory)

$\sigma-$algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions, L_p-spaces, Riesz representation theorem for the space $C[0, 1]$.

Reference reading materials:

1. G. De Barra, "Measure theory and integration".
2. J. Nevue, "Mathematical foundations of the calculus of probability", Holden-Day, Inc., 1965.
3. I. K. Rana, "An introduction to measure and integration", Narosa Publishing House.
4. P. Billingsley, "Probability and measure", John Wiley & Sons, Inc., 1995.
5. W. Rudin, "Real and complex analysis", McGraw-Hill Book Co., 1987.
6. K. R. Parthasarathy, "Introduction to probability and measure", The Macmillan Co. of India, Ltd., 1977.
7. S. Athreya and V. S.Sunder, Measure and Probability, CRC book press, 2018

(Advanced Functional Analysis)

Definition and examples of topological vector spaces (TVS) and locally convex spaces (LCS); Linear operators; Hahn-Banach Theorems for TVS/ LCS (analytic and geometric forms); Uniform boundedness principle; Open mapping theorem; Closed graph thoerem; Weak and weak* vector topologies; Bipolar theorem; dual of LCS spaces; Krein-Milman theorem for TVS; Krien-Smulyan theorem for Banach spaces; Inductive and projective limit of LCS.

Reference reading materials:

1. W. Rudin, "Functional Analysis", Tata McGraw-Hill, 2007.
2. A. P. Robertson, W. Robertson, "Topological Vector Spaces", Cambridge Tracts in Mathematics 53, Cambridge University Press, 1980.
3. J. B. Conway, "A Course in Functional Analysis", Graduates Texts in Mathematics 96, Springer, 2006.

(Nonlinear Analysis)

Calculus in Banach spaces, inverse and multiplicit function theorems, fixed point theorems of Brouwer, Schauder and Tychonoff, fixed point theorems for nonexpansive and set-valued maps, predegree results, compact vector fields, homotopy, homotopy extension, invariance theorems and applications.

Reference reading materials:

1. S. Kesavan, "Nonlinear Functional Analysis", Texts and Readings in Mathematics 28, Hindustan Book Agency, 2004.

(Optimization Theory)

Linear programming problem and its formulation, convex sets and their properties, Graphical method, Simplex method, Duality in linear programming, Revised simplex method, Integer programming, Transportation problems, Assignment problems, Games and strategies, Two-person (non) zero-sum games, Introduction to non-linear programming and techniques.

Reference reading materials:

1. J. K. Strayer, "Linear Programming and its Applications", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
2. P. R. Thie, G. E. Keough, "An Introduction to Linear Programming and Game Theory", John Wiley & Sons, 2008.
3. L. Brickman, "Mathematical Introduction to Linear Programming and Game Theory", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
4. D. G. Luenberger, Y. Ye, "Linear and Nonlinear Programming", International Series in Operations Research & Management Science 116, Springer, 2008.

(Operator Algebras)

Banach algebras/C*–algebras: Definition and examples; Spectrum of a Banach algebra; Gelfand transform; Gelfand-Naimark theorem for commutative Banach algebras/ C*-algebras; Functional calculus for C*-algebras; Positive cone in a C*-algebra; Existance of an approximate identity in a C*-algebra; Ideals and Quotients of a C*-algebra; Positive linear functionals on a C*-algebra; GNS construction. Locally convex topologies on the algebras of bounded operators on a Hilbert space, von-Neumann's bi-commutant theorem; Kaplansky's density theorem. Ruan's characterization of Operator Spaces (if time permites).

Reference reading materials:

1. R. V. Kadison, J. R. Ringrose, "Fundamentals of the Theory of Operator Algebras Vol. I", Graduate Studies in Mathematics 15, American Mathematical Society, 1997.
2. G. K. Pedersen, "C*–algebras and their Automorphism Groups", London Mathematical Society Monographs 14, Academic Press, 1979.
3. V. S. Sunder, "An Invitation to von Neumann Algebras", Universitext, Springer-Verlag, 1987.
4. M. Takesaki, "Theory of Operator Algebras Vol. I", Springer-Verlag, 2002.

(Operator Theory)

Compact operators on Hilbert Spaces. (a) Fredholm Theory (b) Index, C*-algebras - noncommutative states and representations, Gelfand-Neumark representation theorem, Von-Neumann Algebras; Projections, Double Commutant theorem, L^∞ functionalCalculus, Toeplitz operators.

Reference reading materials:

1. W. Arveson, "An invitation to C*-algebras", Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, "Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space", Interscience Publishers John Wiley i& Sons 1963.
3. R. V. Kadison and J. R. Ringrose, "Fundamentals of the theory of operator algebras. Vol. I. Elementary theory", Pure and Applied Mathematics, 100, Academic Press, Inc., 1983.
4. V. S. Sunder, "An invitation to von Neumann algebras", Universitext, Springer-Verlag, 1987.

(Harmonic Analysis)

Fourier series and its convergences, Dirichlet kernel, Frejer kernel, Parseval formula and its applications. Fourier transforms,the Schwartz space, Distribution and tempered distribution, Fourier Inversion and Plancherel theorem. Fourier analysis on L_p-spaces. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem for distribution. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem.

Reference reading materials:

1. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
2. E. M. Stein, G. Weiss, "Introduction to Fourier Analysis on Euclidean Spaces", Princeton Mathematical Series 32, Princeton University Press, 1971.
3. G. B. Folland, "Fourier Analysis and its Applications", Pure and Applied Undergraduate Texts 4, America Mathematical Society, 2010.

(Abstract Harmonic Analysis)

Topological Groups: Basic properties of topological groups, subgroups, quotient groups. Examples of various matrix groups. Connected groups. Haar measure: Discussion of Haar measure without proof on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$ and simple matrix groups, Convolution, the Banach algebra $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$. Basic Representation Theory: Unitary representation of groups, Examples and General properties, The representations of Group and Group algebras, C^*-algebra of a group, GNS construction, Positive definite functions, Schur's Lemma. Abelian Groups: Fourier transform and its properties, Approximate identities in $L^1(G)$, Classical Kernels on $\mathbb{R}$, The Fourier inversion Theorem, Plancherel theorem on $\mathbb{R}$, Plancherel measure on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$. Dual Group of an Abelian Group: The Dual group of a locally compact abelian group, Computation of dual groups for $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$, Pontryagin's Duality theorem.

Reference reading materials:

1. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
2. H. Helson, "Harmonic Analysis", Texts and Readings in Mathematics, Hindustan Book Agency, 2010.
3. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
4. L. H. Loomis, "An Introduction to Abstract Harmonic Analysis", Dover Publication, 2011.
5. E. Hewitt, K. A. Ross, "Abstract Harmonic Analysis Vol. I", Springer-Verlag, 1979.
6. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.

(Lie Algebras)

Definitions and Examples, Derivations, Ideals, Homomorphisms, Nilpotent Lie Algebras and Engel's theorem, Solvable Lie Algebras and Lie's theorem, Jordan decomposition and Cartan's criterion, Semisimple Lie algebras, Casimir operator and Weyl's theorem, Representations of $sl(2, F)$, Root space decomposition, Abstract root systems, Weyl group and Weyl chambers, Classification of irreducible root systems, Abstract theory of weights, Isomorphism and conjugacy theorems, Universal enveloping algebras and PBW theorem, Representation theory of semi-simple Lie algebras, Verma modules and Weyl character formula.

Reference reading materials:

1. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
2. K. Erdmann, M. J. Wildon, "Introduction to Lie Algebras", Springer Undergraduate Mathematics Series, Springer-Verlag, 2006.
3. J.-P. Serre, "Complex Semisimple Lie Algebras", Springer Monographs in Mathematics, Springer-Verlag, 2001.
4. N. Jacobson, "Lie Algebras", Dover Publications, 1979.

(Lie Groups and Lie Algebras - I)

General Properties: Definition of Lie groups, subgroups, cosets, group actions on manifolds, homogeneous spaces, classical groups. Exponential and logarithmic maps, Adjoint representation, Lie bracket, Lie algebras, subalgebras, ideals, stabilizers, center Baker-Campbell-Hausdorff formula, Lie's Theorems. Structure Theory of Lie Algebras: Solvable and nilpotent Lie algebras (with Lie/Engel theorems), semisimple and reductive algebras, invariant bilinear forms, Killing form, Cartan criteria, Jordan decomposition. Complex semisimple Lie algebras, Toral subalgebras, Cartan subalgebras, Root decomposition and root systems. Weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, BGG resolution, Weyl character formula.

Reference reading materials:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
6. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
7. V. S. Varadharajan, "Lie Groups, Lie Algebras and their Representations", Springer-Verlag, 1984.

(Lie Groups and Lie Algebras - II)

General theory of representations, operations on representations, irreducible representations, Schur's lemma, Unitary representations and complete reducibility. Compact Lie groups, Haar measure on compact Lie groups, Schur's Theorem, characters, Peter-Weyl theorem, universal enveloping algebra, Poincare-Birkoff-Witt theorem, Representations of Lie($SL(2,\mathbb{C})$). Abstract root systems, Weyl group, rank 2 root systems, Positive roots, simple roots, weight lattice, root lattice, Weyl chambers, simple reflections, Dynkin diagrams, classification of root systems, Classification of semisimple Lie algebras. Representations of Semisimple Lie algebras, weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, Weyl Character formula, The representation theory of $SU(3)$, Frobenius Reciprocity theorem, Spherical Harmonics.

Reference reading materials:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
6. A. W. Knapp, "Lie Groups: Beyond an introduction", Birkäuser, 2002.
7. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.
8. Brocker and Tom Dieck, Represenations of compact Lie group (Springer-Verlag GTM), 1985.

(Representations of Linear Lie Groups)

Introduction to topological group, Haar measure on locally compact group, Representation theory of compact groups, Peter Weyl theorem, Linear Lie groups, Exponential map, Lie algebra, Invariant Differentail operators, Representation of the group and its Lie algebra. Fourier analysis on $SU(2)$ and $SU(3)$. Representation theory of Heisenberg group . Representation of Euclidean motion group.

Reference reading materials:

1. J. E. Humphreys, "Introduction to Lie algebras and representation theory", Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram, U. B. Tiwari, "A first course on representation theory and linear Lie groups", University Press, 2000.
3. Mitsou Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
4. Sundaram Thangavelu, "Harmonic Analysis on the Heisenberg Group", Birkhauser, 1998.
5. Sundaram Thangavelu, "An Introduction to the Uncertainty Principle", Birkhauser, 2003.

(Harmonic Analysis on Compact Groups)

Review of General Theory: Locally compact groups, Computation of Haar measure on $\mathbb{R}$, $\mathbb{T}$, $SU(2)$, $SO(3)$ and some simple matrix groups, Convolution, the Banach algebra $L^1(G)$. Representation Theory: General properties of representations of a locally compact group, Complete reducibility, Basic operations on representations, Irreducible representations. Representations of Compact groups: Unitarilzibality of representations, Matrix coefficients, Schur's orthogonality relations, Finite dimensionality of irreducible representations of compact groups. Various forms of Peter-Weyl theorem, Fourier analysis on Compact groups, Character of a representation. Schur's orthogonality relations among characters. Weyl's Chracter formula, Computing the Unitary dual of $SU(2)$, $SO(3)$; Fourier analysis on $SO(n)$.

Reference reading materials:

1. T. Brocker, T. Dieck, "Representations of Compact Lie Groups", Springer-Verlag, 1985.
2. J. L. Clerc, "Les Représentatios des Groupes Compacts, Analyse Harmonique" (J. L. Clerc et. al., ed.), C.I.M.P.A., 1982.
3. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
4. M. Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
5. E. B. Vinberg, "Linear Representations of Groups", Birkhäuser/Springer, 2010.
6. A. Wawrzyńczyk, "Group Representations and Special Functions", PWN–Polish Scientific Publishers, 1984.

(Differential Equations)

Ordinary Differential Equations: Initial and boundary value problems, Basic existence, Uniqueness theorems for a system of ODE, Gronwalls lemma, Continuous dependence on initial data, Linear systems with variable coefficients, Variation of parameter formula, Floquet theory, Systems of linear equations with constant coefficients, Stability of equilibrium positions.Partial Differential Equations: Single and systems of PDE, First order PDE, Semi-linear and non-linear equations (Monges method), Four important linear PDE, Transport equations, Laplace equations, Fundamental solution, Mean value formulas, Greens functions, Energy methods, Heat equation, fundamental solution, Mean value formula, Energy methods, Wave equations, Solutions by spherical mean, Energy method, Maximum principle for elliptic and parabolic equations with applications.

Reference reading materials:

1. V. I. Arnold, Ordinary Differential Equations , Prentice Hall of India.
2. Brauer and Nohel, Qualitative Theory of Differential Equations, Dover Publications.
3. Coddington and Levinson, Ordinary Differential Equations, Tata Mcgraw-Hill.
4. Fritz John, Partial Differential Equation, Narosa Publications.
5. M. Renardy and R. C. Rogers, An Introduction to Partial Differential Equations, Springer International Edition.
6. L. C. Evans, Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol 19.

(Partial Differential Equations)

Classification of Partial Differential Equations, Cauchy Problem, Cauchy-Kowalevski Theorem, Lagrange-Green identity, The uniquness theorem of Holmgren, Transport equation: Initial value problem, nonhomogeneous problem. Laplace equation: Fundamental solution, Mean

Value formula, properties of Harmonic functions, Green's function, Energy methods, Harnack's inequality. Heat Equation: Fundamental solution, Mean value formula, properties of solutions. Wave equation: Solution by spherical means, Nonhomogeneous problem, properties of solutions.

Reference reading materials:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. F. John, "Partial Differential Equations", Springer International Edition, 2009.
3. G. B. Folland, "Introduction to Partial Differential Equations", Princeton University Press, 1995.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.

(Advanced Partial Differential Equations)

Distribution Theory, Sobolev Spaces, Embedding theorems, Trace theorem. Dirichlet, Neumann and Oblique derivative problem, Weak formulation, Lax–Milgram, Maximum Principles–Weak and Strong Maximum Principles, Hopf Maximum Principle, Alexandroff-Bakelmann-Pucci Estimate.

Reference reading materials:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Universitext, Springer, 2011.
3. R. A. Adams, J. J. F. Fournier, "Sobolev Spces", Pure and Applied Mathematics 140, Elsevier/Academic Press, 2003.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.
5. M. Renardy, R. C. Rogers, "An Introduction to Partial Differential Equations", Springer, 2008.

(Ergodic Theory)

Measure preserving systems; examples: Hamiltonian dynamics and Liouvilles theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product, Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlins lemma. Recurrence in Topological Dynamics, Birkhoffs Recurrence theorem, Ergodicity, Weak-mixing and strong-mixing and their characterizations, Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyls theorem, The Isomorphism problem; conjugacy, spectral equivalence, Transformations with discrete spectrum, Halmosvon Neumann theorem, Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-MillanBreiman Theorem, Flows. Birkhoffs ergodic Theorem and Wieners ergodic theorem for flows. Flows built under a function.

Reference reading materials:

1. Peter Walters, "An introduction to ergodic theory", Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
2. Patrick Billingsley, "Ergodic theory and information", Robert E. Krieger Publishing Co., 1978.
3. M. G. Nadkarni, "Basic ergodic theory", Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
4. H. Furstenberg, "Recurrence in ergodic theory and combinatorial number theory", Princeton University Press, 1981.
5. K. Petersen, "Ergodic theory", Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

(Probability Theory-I)

Review of Basic undergraduate probability: Random variables, Standard discrete and continuous distributions, Expectation, Variance, Conditional Probability.

Discrete time Markov chains: countable state space, classification of states
Characteristic functions, modes of convergences, Borel-Cantelli Lemma, Central Limit Theorem, Law of Large numbers, Convergence Theorems in Markov Chains.

Reference reading materials:

1. W. Feller: Introduction to Probability Theory and ins Applications Vol.I & Vol. II (Wiley)
2. S. M. Ross: Introduction to Probability Models (AP)
3. Hoel, Port & Stone: Introduction to Stochastic Processes (HMC)
4. S. M. Ross: Stochastic Processes (Wiley)

(Probability Theory - II)

Martingale Theory: Radon-Nikoydm Theorem, Doob-Meyer decomposition.
Weak convergence of probability measures,
Brownian motion, Markov processes and Stationary processes.

Reference reading materials:

1. O. Kallenberg: Foundation of Modern Probability (Springer)
2. P. Billingsley: Convergence of probability measures.(John Wiley & Sons, Inc.)
3. D. Revuz & M. Yor: Continuous martingales and Brownian motion (Springer-Verlag)
4. S. M. Ross: Stochastic Processes (Wiley)
5. J. L. Doob: Stochastic Processes (Wiley)

(Advanced Probability)

Probability spaces, Random Variables, Independence, Zero-One Laws, Expectation, Product spaces and Fubini's theorem, Convergence concepts, Law of large numbers, Kolmogorov three-series theorem, Levy-Cramer Continuity theorem, CLT for i.i.d. components, Infinite Products of probability measures, Kolmogorov's Consistency theorem, Conditional expectation, Discrete parameter martingales with applications.

Reference reading materials:

1. A. Gut, "Probability: A Graduate Course", Springer Texts in Statistics, Springer, 2013.
2. K. L. Chung, "A Course in Probability Theory", Academic Press, 2001.
3. S. I. Resnick, "A Probability Path", Birkhäuser, 1999.
4. P. Billingsley, "Probability and Measure", Wiley Series in Probability and Statistics, John Wiley & Sons, 2012.
5. J. Jacod, P. Protter, "Probability Essentials", Universitext, Springer-Verlag, 2003.

(Mathematical Foundations for Finance)

Financial market models in finite discrete time, Absence of arbitrage and martingale measures, Valuation and hedging in complete markets, Basic facts about Brownian motion, Stochastic integration, Stochastic calculus: Itô's formula, Girsanov transformation, Itô's representation theorem, Black-Scholes formula

Reference reading materials:

1. J. Jacod, P. Protter, "Probability Essentials", Universitext, Springer-Verlag, 2003.
2. D. Lamberton, B. Lapeyre, "Introduction to Stochastic Calculus Applied to Finance", Chapman-Hall, 2008.
3. H. Föllmer, A. Schied, "Stochastic Finance: An Introduction in Discrete Time", de Gruyter, 2011.

(Brownian Motion and Stochastic Calculus)

Brownian Motion, Martingale, Stochastic integrals, extension of stochastic integrals, stochastic integrals for martingales, Itô's formula, Application of Itô's formula, stochastic differential equations.

Reference reading materials:

1. H. H. Kuo, "Introduction to Stochastic Integration", Springer, 2006.
2. J. M Steele, "Stochastic Calculus and Financial Applications", Springer-Verlag, 2001.
3. F. C. Klebaner, "Introduction to Stochastic Calculus with Applications", Imperial College, 2005.

(Randomized Algorithms and Probabilistic Methods)

Inequalities of Markov and Chebyshev (median algorithm), first and second moment method (balanced allocation), inequalities of Chernoff (permutation routing) and Azuma (chromatic number), rapidly mixing Markov chains (random walk in hypercubes, card shuffling), probabilistic generating functions (random walk in d-dimensional lattice)

Reference reading materials:

1. R. Motwani, P. Raghavan, "Randomized Algorithms", Cambridge University Press, 2004.
2. M. Mitzenmacher, E. Upfal, "Probability and Computing: Randomized algorithms and probabilistic analysis", Cambridge University Press, 2005.

(Introduction to Stochastic Processes)

Discrete Markov chains with countable state space; Classification of states: recurrences, transience, periodicity. Stationary distributions, reversible chains, Several illustrations including the Gambler's Ruin problem, queuing chains, birth and death chains etc. Poisson process, continuous time Markov chain with countable state space, continuous time birth and death chains.

Reference reading materials:

1. P. G. Hoel, S. C. Port, C. J. Stone, "Introduction to Stochastic Processes", Houghton Mifflin Co., 1972.
2. R. Durrett, "Essentials of Stochastic Processes", Springer Texts in Statistics, Springer, 2012.
3. G. R. Grimmett, D. R. Stirzaker, "Probability and Random Processes", Oxford University Press, 2001.
4. S. M. Ross, "Stochastic Processes", Wiley Series in Probability and Statistics: Probability and Statistics, John Wiley & Sons, 1996

(Statistical Inference I)

Review: joint and conditional distributions, order statistics, group family, exponential family. Introduction to parametric inference, sufficiency principle and data reduction, factorization theorem, minimal sufficient statistics, Fisher information, ancillary statistics, complete statistics, Basu's theorem. Unbiasedness, best unbiased and linear unbiased estimator, Rao-Blackwell theorem, Lehmann- Scheffe theorem and UMVUE, Cramer-Rao lower bound and UMVUE, multi-parameter cases. Location and scale invariance, principle of equivarience. Methods of estimation: method of moments, likelihood principle and maximum likelihood estimation, properties of MLE: invariance, consistency, asymptotic normality. Hypothesis testing: error probabilities and power, most powerful tests, Neyman-Pearson lemma and its applications, p-value, uniformly most powerful (UMP) test via Neyman- Pearson lemma, UMP test via monotone likelihood ratio property, existence and nonexistence of UMP test for two sided alternative, unbiased and UMP unbiased tests. Likelihood (generalized) ratio tests and its properties, invariance and most powerful invariant tests. Introduction to confidence interval estimation, methods of fining confidence intervals: pivotal quantity, inversion of a test, examples such as confidence interval for mean, variance, difference in means, optimal interval estimators, uniformly most accurate confidence bound, large sample confidence intervals.

Reference reading materials:

1. E. L. Lehmann and G. Casella, "Theory of Point Estimation", 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann and J. P. Romano, "Testing Statistical Hypothesis", 3rd edition, Springer, 2005.
3. N. Mukhopadhyay, "Probability and Statistical Inference", Marcel Dekker, New York. 2000.
4. G. Casella and R. L. Berger, "Statistical Inference", 2nd edition, Cengage Learning, 2001.

5. A. M. Mood, F. A. Graybill and D. C. Boes, "Introduction to the theory of Statistics", 3rd edition, McGraw Hill, 1974.

(Statistical Inference II)

General decision problem, loss and risk function, minimax estimation, minimaxity and admissibility in exponential family. Introduction to Bayesian estimation, Bayes rule as average risk optimality, prior and posterior, conjugate families, generalized Bayes rules. Bayesian intervals and construction of credible sets, Bayesian hypothesis testing. Empirical and nonparametric empirical Bayes analysis, admissibility of Bayes and generalized Bayes rules, discussion on Bayes versus non-Bayes approaches. Large sample theory: review of modes of convergences, Slutsky's theorem, Berry-Essen bound, delta method, CLT for iid and non iid cases, multivariate extensions. Asymptotic level α tests, asymptotic equivalence, comparison of tests: relative efficiency, asymptotic comparison of estimators, efficient estimators and tests, local asymptotic optimality. Bootstrap sampling: estimation and testing.

Reference reading materials:

1. E. L. Lehmann and G. Casella, "Theory of Point Estimation", 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann, "Elements of Large-Sample Theory", Springer-Verlag, 1999.
3. E. L. Lehmann and J. P. Romano, "Testing Statistical Hypothesis", 3rd edition, Springer, 2005.
4. James O Berger, "Statistical Decision Theory and Bayesian Analysis", 2nd edition, Springer, New York, 1985.

(Multivariate Statistical Analysis)

Review of matrix algebra (optional), data matrix, summary statistics, graphical representations. Distribution of random vectors, moments and characteristic functions, transformations, some multivariate distributions: multivariate normal, multinomial, Dirichlet distribution, limit theorems. Multivariate normal distribution: properties, geometry, characteristics function, moments, distributions of linear combinations, conditional distribution and multiple correlation. Estimation of mean and variance of multivariate normal, theoretical properties, James-Stein estimator (optional), distribution of sample mean and variance, the Wishart distribution, large sample behavior of sample mean and variance, assessing normality. Inference about mean vector: testing for normal mean, Hotelling T^2 and likelihood ratio test, confidence regions and simultaneous comparisons of component means, paired comparisons and a repeated measures design, comparing mean vectors from two populations, MANOVA. Techniques of dimension reduction, principle component analysis: definition of principle components and their estimation, introductory factor analysis, multidimensional scaling. Classification problem: linear and quadratic discriminant analysis, logistic regression, support vector machine. Cluster analysis: non-hierarchical and hierarchical methods of clustering.

Reference reading materials:

1. K. V. Mardia, J. T. Kent and J. M. Bibby, "Multivariate Analysis", Academic Press, 1980.
2. T. W. Anderson, "An introduction to Multivariate Statistical Analysis", Wiley, 2003.
3. C. Chatfield and A. J. Collins, "Introduction to Multivariate Analysis", Chapman & Hall, 1980.
4. R. A. Johnson and D. W. Wichern, "Applied Multivariate Statistical Analysis", 6th edition, Pearson, 2007.
5. Brian Everitt and Torsten Hothorn, "An Introduction to Applied Multivariate Analysis with R", Springer, 2011.
6. M. L. Eaton, "Multivariate Statistics", John Wiley, 1983.

(Regression Analysis)

Introduction to simple linear regression, least square estimation and hypothesis testing of model parameters, prediction, interval estimation in simple linear regression, Coefficient of determination, estimation by maximum likelihood, multiple linear regression, matrix representation of the regression model, estimation and testing of model parameters and prediction, model adequacy checking-residual analysis, PRESS statistics, outlier detection, lack of fit test, serial correlation and Durbin-Watson test, transformation and weighting to correct model inadequacies-variance-stabilizing transformation, generalized and weighted least squares, diagnostics for influential observations, Cooks D test, multicollinearity-sources and effects, diagnosis and treatment for multicollinearity, ridge regression and LASSO, bootstrap estimation, dummy variable model, variable selection and model buildingstepwise methods, polynomial regression and interaction regression models, nonlinear regression, generalized linear models-logistic regression and Poisson regression.

Reference reading materials:

1. Douglas C. Montgomery, Elizabeth A. Peck, G. Geoffrey Vining, Introduction to Linear Regression Analysis, 5th Edition, Wiley, 2012.
2. N. R. Draper and H. Smith (1998), Applied Regression Analysis, 3rd Edition, New York: Wiley.
3. Michael H. Kutner, Chris J. Nachtsheim, and John Neter, Applied Linear Statistical Models, McGraw-Hill/Irwin; 5th edition, 2004.
4. Seber, G. A. F. and Lee, A. J., Linear Regression Analysis, John Wiley and Sons, 2nd Edition, 2003.
5. N. H. Bingham, John M. Fry, Regression: Linear Models in Statistics, Springer Undergraduate Mathematics Series, 2010.

(Time Series Analysis)

Examples and objectives of time series, stationary time series and autocorrelation function, estimation and elimination of trend and seasonal components, testing for noise sequence, moving average process, autoregressive processes and ARMA processes, estimation of autocorrelation function, methods of forecasting-Durbin-Levinson algorithm and Innovations algorithm, the Wold decomposition, ARMA models-the auto-covariance and partial auto-covariance function, forecasting ARMA processes, spectral analysis-spectral densities, periodogram, modeling with ARMA processes, Yule-Walker estimation, maximum likelihood estimation, diagnostic checking, non-stationary time series-ARIMA models, identification techniques, forecasting ARIMA models, seasonal ARIMA models, multivariate time series, ARCH and GARCH models.

Reference reading materials:

1. Peter J. Brockwell and Richard A. Davis, Introduction to Time Series and Forecasting, Springer Texts in Statistics, 2010.
2. Chris. Chatfield, The analysis of time series: An introduction, 6th edition, Chapman & Hall/CRC, 2004.
3. Michael H. Kutner, Chris J. Nachtsheim, and John Neter, Applied Linear Statistical Models, McGraw-Hill/Irwin; 5th edition, 2004.
4. J. D. Cryer and K.-S. Chan, Time series analysis with applications in R, 2nd edition, Springer, 2008.
5. R. H. Shumway and D. S. Stoffer, Time series analysis and its applications with R examples, 3rd edition, Springer, 2011.